Geometric BRST Quantization, I: Prequantization

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Abstract. This is the first part of a two-part paper dedicated to the definition of BRST quantization in the framework of geometric quantization. After recognizing prequantization as a manifestation of the Poisson module structure of the sections of the prequantum line bundle, we define BRST prequantization and show that it is the homological analog of the symplectic reduction of prequantum data. We define a prequantum BRST cohomology theory and interpret it in terms of geometric objects. We then show that all Poisson structures correspond under homological reduction. This allows to prove, in the BRST context, that prequantization and reduction commute.

1. Introduction

This is the first part of a two-part paper aimed at defining BRST quantization. Although BRSTquantization has become the preferred method to quantize gauge systems, almost no attention has been focused on the problem of defining this procedure formally nor to justify its validity; the only justification for the validity of the BRST quantization procedure being that it is analogous to its better-understood classical counterpart. The BRST quantization of a gauge theory consists roughly in the quantization of a larger system to which it corresponds classically (after homological reduction). However only in very special systems (e.g., free string theory) can one actually show that the quantum theories also correspond (after homological reduction).

We shall work throughout this paper in the symplectic or hamiltonian framework in which classical BRST appears in its more natural form. To fix the ideas, let (M, Ω) be a symplectic manifold in which one has defined a set of irreducible first class constraints. Then there is a geometric construction (outlined in Sect. 2)

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