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Shift of the Shadow Boundary in High Frequency Scattering

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Abstract. The microlocal theory of diffraction is used to establish the conjecture of Keller and Rubinow relating the shift of the shadow boundary in high frequency scattering to the directional curvatures of a strictly convex obstacle.

1. Introduction and Statement of Results

In their paper on scattering by a cylinder [3], Keller and Rubinow conjecture that the shift of the shadow cast by any object is asymptotically equal to

$$C\alpha^{-1/3}\lambda^{-2/3},$$

along each glancing ray, where λ is the frequency and α is the directional curvature of the object at the glancing point at the direction of the glancing ray. The constant C depends only on the boundary condition at the glancing point and is positive for the Dirichlet problem (hard obstacle), negative for the Neumann problem (soft obstacle), i.e. the shadow boundary is shifted outwards or inwards respectively.

The work of Keller and Rubinow followed earlier computations of Artmann, Rice and Logan. For scattering by a sphere in $\mathbf{R}^3$, the conjecture was proved by Nussenzveig [10].

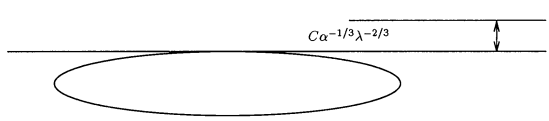

Fig. 1

The purpose of this note is to establish the asymptotic shift of the shadow boundary for scattering by any strictly convex obstacle in $\mathbf{R}^n$. This is done using