

The Higher Rank Virasoro Algebras*

J. Patera¹ and H. Zassenhaus²

¹ Centre de Recherches mathématiques, Université de Montréal, CP 6128-A, Montréal (Québec) H3C 3J7, Canada

² Department of Mathematics, Ohio State University, Columbus, Ohio 43210, USA

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Abstract. Higher rank Virasoro algebras are defined and their properties studied: triangular decompositions, automorphism groups and finite dimensional subalgebras.

In the following paper we generalize what has recently been called the Virasoro algebra. We reluctantly continue to call that algebra the Virasoro algebra, since in [1] the algebra appears (only implicitly) without the central extension. Such an algebra was well known to E. Cartan [2] and was extensively studied [3, 4, 5] before World War II, on a suggestion of E. Witt, as an example of a simple Lie algebra (of infinite dimension over \mathbb{C} and of finite dimension over \mathbb{F} of char > 0). With the central extension added, the algebra is not simple but its representation theory is much richer. The central term apparently appeared for the first time in [6] with a footnote reference to J. Weis.

The purpose of this article is to generalize the notion of Virasoro algebras of rank 1 to higher rank Virasoro algebras and explore its algebraic properties like triangular decompositions, the automorphism groups and the finite dimensional subalgebras. Six representative examples are described in the last section. A comprehensive exposition of the rank one Virasoro algebra and its representations is found in [7].

1. Definitions

In [2] the infinite simple Lie algebra $L(\mathbb{Z}/\mathbb{C})$ with basis elements $e_j (j \in \mathbb{Z})$ and multiplication rule

$$[e_j, e_k] = (k - j)e_{j+k} \quad (1)$$

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