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Scaling Limit for Interacting Ornstein-Uhlenbeck Processes *

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Abstract. The problem of describing the bulk behavior of an interacting system consisting of a large number of particles comes up in different contexts. See for example [1] for a recent exposition. In [4] one of the authors considered the case of interacting diffusions on a circle and proved that the density of particles evolves according to a nonlinear diffusion equation. The interacting particles evolved according to a generator that was symmetric in equilibrium. In this article we consider interacting Ornstein-Uhlenbeck processes. Here the diffusion generator is not symmetric relative to the equilibrium and the earlier methods have to be modified considerably. We use some ideas that were employed in [3] to extend the central limit theorem from the symmetric to nonsymmetric cases.

1. The Model and Its Macroscopic Equation

Let S be the circle of circumference 1. For each positive integer N we consider a system of N interacting particles with positions on S and velocities in R. The system is described by the following stochastic differential equations in phase space $(\underline{x}, \underline{v}) = \{(x_1, v_1), (x_2, v_2), \ldots, (x_N, v_N)\}$,

$$\begin{aligned} dx_i(t) &= N v_i(t)\,dt \\ dv_i(t) &= -N^2 \sum_{j \neq i} 2V'(N(x_i(t) - x_j(t)))\,dt \\ &\quad - \frac{N^2}{2} v_i(t)\,dt + N\,dw_i(t) \end{aligned} \tag{1.1}$$

for $i = 1, 2, \ldots, N$; $0 \leqq t \leqq T$. Here $\{w_i(t), i = 1, 2, \ldots, N\}$ are N independent Wiener processes and V is an even function on R with compact support describing a pair interaction.

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