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Scaling Limits for Interacting Diffusions*

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Abstract. We consider a large number of particles diffusing on a circle interacting through a drift resulting from the gradient of a pair potential whose support is of the order of the interparticle distance. We derive a nonlinear bulk diffusion equation for the density of the particle distribution on the circle. The diffusion coefficient is determined as a function of density in terms of standard thermodynamical objects.

1. Introduction

In this article we study the hydrodynamic limit for interacting Brownian motions on the one-dimensional circle. The interaction is between pairs of particles and is repulsive in nature. The scaling is such that the range of the interaction is of the same order as interparticle distance and therefore each particle interacts with only a finite number of nearby particles at any given time.

We obtain a nonlinear bulk diffusion equation and the diffusion coefficient is naturally expressed in terms of the thermodynamic functions of our one dimensional system. The main limitations are the finiteness of volume that is forced because our basic space is the circle, the repulsive nature of the interactions that is assumed and that we are in one space dimension.

In this context the fluctuations around equilibrium have been studied earlier by H. Spohn [5] and the self diffusion in equilibrium by Guo and Papanicolaou [1]. Their results have been derived for infinite volume and arbitrary space dimension. Of course these results deal essentially with equilibrium problems where a lot of control through estimates is available.

Our method is similar to the one used in Guo, Papanicolaou and Varadhan [2] and uses estimates based on entropy and its rate of change. Section 2 describes

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