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Quantization of the Poisson $SU(2)$ and Its Poisson Homogeneous Space – The 2-Sphere*

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Abstract. We show that deformation quantizations of the Poisson structures on the Poisson Lie group $SU(2)$ and its homogeneous space, the 2-sphere, are compatible with Woronowicz's deformation quantization of $SU(2)$'s group structure and Podles' deformation quantization of 2-sphere's homogeneous structure, respectively. So in a certain sense the multiplicativity of the Lie Poisson structure on $SU(2)$ at the classical level is preserved under quantization.

Introduction

In the area of quantization of (symplectic manifolds), there have been two major approaches, namely geometric quantization and deformation quantization. In this paper, we shall work with the second approach, which seems to be more realistic physically, although the first approach is mathematically beautiful and intriguing. In the seventies, Bayen, Flato, Fronsdal, Lichnerowicz and Sternheimer first formalized the concept of deformation quantization of the symplectic (or Poisson) structure of a manifold in terms of formal power series [Ba–Fl–Fr–Li–St]. Since then there has been a lot of research in this direction. Recently, Marc A. Rieffel formulated such a theory in the context of C^*-algebras and obtained interesting results [Ri1, 2, 3]. From a certain point of view, this formulation has the advantage of being closer to the traditional way of quantization using operators on Hilbert spaces.

Parallel to the above quantization of geometric structures, there is a theory of deformation quantization of group structures, namely the theory of quantum groups [Dr]. Also recently, S. L. Woronowicz developed such a theory in the

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