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# Semiclassical Yang-Mills Theory I: Instantons

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**Abstract.** The partition functions of quantum Yang-Mills theory have an expansion in powers of the coupling constant; the leading order term in this expansion is called the semiclassical approximation. We study the semiclassical approximation for Yang-Mills theory on a compact Riemannian 4-manifold using geometric techniques, and do explicit calculations for the case when the manifold is the 4-sphere. This involves calculating the Riemannian measure and certain functional determinants on the moduli space of self-dual connections. The main result is that the contribution to the semiclassical partition functions coming from the $k=1$ connections on the 4-sphere is *finite* and *calculable.* We also discuss a renormalization procedure in which the radius of the 4-sphere is allowed to tend to infinity.

## 0. Introduction

In previous articles ([GP 1, GP 2, and Gr]) the authors have described the Riemannian geometry of the moduli space of self-dual connections on compact 4-manifolds. In this paper we extend those results to address a question of more direct physical interest: the geometry of the semiclassical approximation to the partition functions for Yang-Mills theories on such manifolds. From a geometric perspective these semiclassical approximations arise as follows.

Given a principal $G$-bundle $P$ over a compact oriented Riemannian 4-manifold $(M, g)$, let $\mathscr{A}_P$ and $\mathscr{G}_P$ be the space of connections and the gauge group of $P$. Let $\mathscr{A} = \mathscr{A}(M)$ be the disjoint union of the $\mathscr{A}_P$ over all equivalence classes of bundles $P \to M$. The quantum expectation of a gauge-invariant function $\Phi : \mathscr{A} \to \mathbf{R}$ is defined formally as a quotient of two integrals over $\mathscr{A}$:

$$\langle \Phi \rangle = \frac{\int_{\mathscr{A}} \Phi(A) e^{-S} d\mathscr{A}}{\int_{\mathscr{A}} e^{-S} d\mathscr{A}} = \frac{Z(\Phi)}{Z(1)}. \tag{0.1}$$

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