Commun. Math. Phys. 135, 79–89 (1990)

Communications in
Mathematical
Physics

Gibbs Measure as Quantum Ground States

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Received March 22, 1990

Abstract. We study certain quantum spin systems which are equivalent to stochastic Ising models. We show that any translationally invariant quantum ground state is given by integration of Gibbs measure. The existence of mass gap is shown to be the same as exponentially fast convergence of stochastic models to invariant states.

1. Introduction

In [6], we introduce a class of quantum spin systems using translationally invariant ground states. The aim of this paper is to extend our results to a wider class of potentials.

Our new idea is as follows. Assuming certain convergence of finite ground state vectors, we rewrite the Hamiltonian in the form of a generator of a Markov semi-group on the classical spin system.

The ground state property is almost equivalent to the invariance of a measure with respect to the associated Markov semigroup. The proof is rather algebraic. We also prove the existence of the gap of the spectrum of quantum Hamiltonian using results of stochastic Ising models. We use a C^* algebraic approach. (See [2] for the basics.)

We first introduce some notations. We consider the algebra of observables A which is the UHF C^* algebra,

$$A = \bigotimes_{\mathbb{Z}^d} M_2(C). \tag{1.1}$$

So A is generated by Pauli spin matrices $\sigma_\alpha^{(j)}$ ($\alpha = x, y, z$) on j^{th} site. $\sigma_\alpha^{(j)}$ is a selfadjoint unitary satisfying

$$[\sigma_\alpha^{(j)}, \sigma_\beta^{(k)}] = \sigma_\alpha^{(j)}\sigma_\beta^{(k)} - \sigma_\beta^{(k)}\sigma_\alpha^{(j)} = 0, \tag{1.2a}$$

$$\sigma_\alpha^{(j)}\sigma_\beta^{(j)} = i\varepsilon_{\alpha\beta\gamma}\sigma_\gamma^{(j)}, \tag{1.2b}$$

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