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Generic Global Solutions of the Relativistic Vlasov–Maxwell System of Plasma Physics

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Abstract. The behaviour of classical solutions of the relativistic Vlasov–Maxwell system under small perturbations of the initial data is investigated. First it is shown that the solutions depend continuously on the initial data with respect to various norms. The main result is on global solutions: A global solution whose electromagnetic field decays in a certain way for large times is shown to remain global under small perturbations of the initial data and to retain the decay behaviour of the field. Therefore, such global solutions are generic. This result implies the existence of global solutions for nearly symmetric initial data.

1. Introduction

Consider a collisionless plasma with N different species of particles, where a particle of species α has rest mass m_α and charge e_α. Each species is described by a particle density $f_\alpha(t,x,v)$, where $t \geqq 0$ denotes time, $x \in \mathbb{R}^3$ position, and $v \in \mathbb{R}^3$ momentum. The particles may move at relativistic speeds and are assumed to interact only by the electromagnetic forces they create themselves so that the density functions $(f_\alpha)_{\alpha=1}^N = f$ together with the selfconsistent electromagnetic fields E_f and B_f evolve according to the relativistic Vlasov–Maxwell system (RVM):

$$\partial_t f_\alpha + \hat{v}_\alpha \cdot \partial_x f_\alpha + e_\alpha (E_f + \hat{v}_\alpha \times B_f) \cdot \partial_v f_\alpha = 0, \qquad 1 \leqq \alpha \leqq N,$$

$$\partial_t E_f - \operatorname{curl} B_f = -4\pi j_f, \quad \operatorname{div} E_f = 4\pi \rho_f,$$

$$\partial_t B_f + \operatorname{curl} E_f = 0, \qquad \operatorname{div} B_f = 0.$$

Here

$$\rho_f(t,x) := \sum_{\alpha=1}^N e_\alpha \int f_\alpha(t,x,v)\,dv$$

and

$$j_f(t,x) := \sum_{\alpha=1}^N e_\alpha \int \hat{v}_\alpha f_\alpha(t,x,v)\,dv$$