Similarity Between the Mandelbrot Set and Julia Sets

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Abstract. The Mandelbrot set $M$ is “self-similar” about any Misiurewicz point $c$ in the sense that if we examine a neighborhood of $c$ in $M$ with a very powerful microscope, and then increase the magnification by a carefully chosen factor, the picture will be unchanged except for a rotation. The corresponding Julia set $J_c$ is also “self-similar” in the same sense, with the same magnification factor. Moreover, the two sets $M$ and $J_c$ are “similar” in the sense that if we use a very powerful microscope to look at $M$ and $J_c$, both focused at $c$, the structures we see look like very much the same.

1. Introduction

For a quadratic polynomial $f_c: z \mapsto z^2 + c$, the filled-in Julia set $K_c$ of $f_c$ is the set of non-escaping points under iteration:

$$K_c = \{ z \in \mathbb{C} | (f_c^n(z))_{n \in \mathbb{N}} \text{ is bounded} \},$$

where $f_c^n$ denotes the $n$th iteration $f_c \circ f_c \circ \ldots \circ f_c$ of $f$. The Julia set of $f_c$ is $J_c = \partial K_c$.

The Mandelbrot set is

$$M = \{ c \in \mathbb{C} | 0 \in K_c \}.$$

One can generate easily the pictures of Julia sets and the Mandelbrot set by computers. Figure 1 is a picture of $M$, Fig. 3a–3d are pictures of $J_c$ for various values of $c$. Globally, $J_c$ and $M$ have completely different shapes. However, their local structures are sometimes very similar. Figure 2 consists of three successive enlargements of $M$ in a neighborhood of $i$. A remarkable resemblance with the Julia set for $c = i$ (Fig. 3a) appears. In fact, this kind of similarity happens for every value of $c$ which is a Misiurewicz point, that is, for which the point 0 under $f_c$ is not

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