Commun. Math. Phys. 134, 555–585 (1990)

Communications in Mathematical Physics

Isospectral Hamiltonian Flows in Finite and Infinite Dimensions

II. Integration of Flows*

M. R. Adams[1], J. Harnad[2] and J. Hurtubise[3]

[1] Department of Mathematics, University of Georgia, Athens, GA 30602, USA
[2] Centre de Recherches Mathématiques, Université de Montréal, C. P. 6128-A, Montréal, Qué, H3C 3J7, Canada and Department of Mathematics, Concordia University, Montréal, Qué, Canada
[3] Department of Mathematics, McGill University, Montréal, Qué, H3A 2K6, Canada

Received May 30, 1989; in revised form March 12, 1990

Abstract. The approach to isospectral Hamiltonian flow introduced in part I is further developed to include integration of flows with singular spectral curves. The flow on finite dimensional Ad*-invariant Poisson submanifolds of the dual $(\widetilde{gl}(r)^+)^*$ of the positive part of the loop algebra $\widetilde{gl}(r)$ is obtained through a generalization of the standard method of linearization on the Jacobi variety of the invariant spectral curve S. These curves are embedded in the total space of a line bundle $T \to \mathbb{P}_1(\mathbb{C})$, allowing an explicit analysis of singularities arising from the structure of the image of a moment map $\tilde{J}_r: M_{N,r} \times M_{N,r} \to (\widetilde{gl}(r)^+)^*$ from the space of rank-r deformations of a fixed $N \times N$ matrix A. It is shown how the linear flow of line bundles $E_t \to \tilde{S}$ over a suitably desingularized curve $\tilde{S}$ may be used to determine both the flow of matricial polynomials $L(\lambda)$ and the Hamiltonian flow in the space $M_{N,r} \times M_{N,r}$ in terms of θ-functions. The resulting flows are proved to be completely integrable. The reductions to subalgebras developed in part I are shown to correspond to invariance of the spectral curves and line bundles $E_t \to \tilde{S}$ under certain linear or anti-linear involutions. The integration of two examples from part I is given to illustrate the method: the Rosochatius system, and the CNLS (coupled non-linear Schrödinger) equation.

Introduction

In [1] it was shown how isospectral Hamiltonian flows in the space of rank r perturbations, $\mathcal{M}_A$, of an $N \times N$ matrix A can be derived from the Adler–Kostant–

* Research supported in part by the Natural Sciences and Engineering Research Council of Canada and by U.S. Army grant DAA L03-87-K-0110