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Virasoro Model Space

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Abstract. The representations of a compact Lie group G can be studied via the construction of an associated "model space." This space has the property that when geometrically quantized its Hilbert space contains every irreducible representation of G just once. We construct an analogous space for the group $\mathrm{Diff}\,S^1$. It is naturally a complex manifold with a holomorphic, free action of $\mathrm{Diff}\,S^1$ preserving a family of pseudo-Kahler structures. All of the "good" coadjoint orbits are obtained from our space by Hamiltonian constraint reduction. We briefly discuss the connection to the work of Alekseev and Shatashvili.

1. Introduction

A geometrical understanding of the representation theory of the group of diffeomorphisms of the circle remains a desirable, and elusive, goal. Apart from its intrinsic interest a solution of this problem could shed light on a $2+1$-dimensional topological quantum field theory standing in the same relation to Virasoro as compact Chern–Simons–Witten theory does to Kac–Moody algebras [1]. Given the success of the method of orbits in understanding the representations of noncompact groups (see e.g. [2]), it is very natural to look to this method for help with $\mathrm{Diff}\,S^1$ as well. Considerable progress has been made along these lines [3], but some problems stand out.

First, there are a variety of different types of orbit. Secondly, while every orbit has naturally the structure of a Hamiltonian dynamical system, there is in general no obvious choice of the additional structures needed to quantize these classical systems. Finally, once a quantization is chosen we find ourselves faced with a strongly-coupled system unless the central charge $c \gg 1$. In the latter case Witten has shown that indeed the familiar irreducible representations emerge.

Clearly it would be interesting to have an approach to this problem where all

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