Commun. Math. Phys. 134, 413–419 (1990)

Communications in
Mathematical
Physics

Deformations and Renormalisations of W_∞

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Received April 3, 1990

Abstract. Deformations of the infinite N limit of the Zamolodchikov W_N algebra are discussed. A recent one, due to Pope, Romans and Shen with non-zero central extensions for every conformal spin is shown to be formally renormalisable to one representable in Moyal bracket form. Another deformation is discovered which, like the algebra of Pope et al. possesses automatic closure, but has non-zero central extension only in the Virasoro subalgebra.

In recent months there has been a great deal of interest in infinite-dimensional algebras which represent area preserving diffeomorphisms of various two-dimensional manifolds [1–3]. These algebras all possess a Poisson structure, and it is a current topic of great activity to extend these considerations to deformations of this Poisson structure. For example, the well-known Moyal bracket deformation was resurrected in the context of the algebra describing area preserving maps on a torus [2] as a means of relating this to the algebra of $SU(N)$ as $N\to\infty$. The idea was to replaced the structure constants of the algebra

$$[L_{j,m}, L_{k,n}]=(mk-nj)L_{j+k,m+n}+(aj+bm)\delta_{j+k,0}\delta_{m+n,0} \tag{1a}$$

by

$$[K_{j,m}, K_{k,n}]=\frac{i}{\lambda}\sin\lambda(mk-nj)K_{j+k,m+n}+(aj+bm)\delta_{j+k,0}\delta_{m+n,0}\,. \tag{1b}$$

The structure function which arises here is just a special case of the Moyal bracket [4] (see also [1]), acting on functions f, g of x, y, which is given by

$$\sin(\lambda\{f,g\})=\sum_{p=0}^{\infty}(-1)^p\frac{\lambda^{2p+1}}{(2p+1)!}\sum_{k=0}^{2p+1}(-1)^k\binom{2p+1}{k} \times(\partial_x^k\partial_y^{2p+1-k}f)(\partial_x^{2p+1-k}\partial_y^k g)\,. \tag{2}$$

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* On Research Leave from the University of Durham UK; research supported in part by the Department of Energy under Grant DE[FG02/88/ER25065, and by a grant from the Alfred P. Sloan Foundation

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