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Self-Dual Yang-Mills Fields and Deformations of Algebraic Curves

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Abstract. Recently it has been shown that the methods of algebraic geometry first used for finding periodic and almost periodic solutions of KdV, HSh, SG and other equations [11–13] may be successfully applied to study the solutions of nonlinear equations with a variable spectral parameter in associated zero-curvature representation. In this work following [20] this treatment is extended to the case of the self-duality equation. It seems to be the first example of a four-dimensional non-linear equation solvable by the method of finite-gap integration. Two broad classes of finite-gap solutions for each – $SU(2)$ and $SU(1,1)$ gauge groups are constructed in terms of multidimensional theta-functions. The dynamics of the solutions is given by the movement of the hyperelliptic curve with moving branch points and a divisor of the poles in the moduli space of algebraic curves. In the general case our solutions have no periodicity property. We show how one-instanton solution and $5N$-parametric t'Hooft family of instantons may be obtained by the degeneration of general formulae.

1. Introduction

The problem of obtaining and investigating particle-like (soliton) solutions of the field equations attracted in the 70's a great interest of many mathematicians and physicists. It is impossible to give here even a brief review of the main results in the field (for history and references see [1–3]); we will mention only results closely related to the subject of this paper.

Belavin and Zakharov [4] obtained a U–V pair for the self-duality equation and, therefore, it appeared possible to apply the inverse scattering technique to this equation. However, one of the most important problems in the field – the description of all multi-instanton configurations was solved in a remarkable paper [5] by a different algebraic-geometric AHDM method. Nahm's modification of this method allowed to solve another problem – the classification of all multimonopole configurations. Despite very good results of this method some important problems remain unsolved – for example, effective description of the