Commun. Math. Phys. 134, 347–370 (1990)

Communications in Mathematical Physics

Quantum and Classical Pseudogroups. Part I. Union Pseudogroups and Their Quantization

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Received December 14, 1989

Abstract. Union pseudogroups (structures analogical to pseudogroups in the sense of [1]) are defined using the category dual to the category of groupoids instead of the category of pseudospaces in the sense of [2]. It is shown that these structures are equivalent to double groups (in the sense of [3]). Moreover, it is shown that a quantization procedure associates with each finite union pseudogroup a (quantum) pseudogroup. Therefore for each finite double group there is a finite pseudogroup.

Introduction

Many attempts have been made in order to create a theory of objects more general than groups, essentially by considering not necessarily commutative spaces. Those efforts are valuable because of the following two features of such generalizations:
1) a principle of duality, which, in particular, can be applied to noncommutative groups,
2) a description of new kind of symmetries.

In spite of the same fundamental idea, there exist many different approaches using even completely different words denoting the objects generalizing groups, such as pseudogroups [1], quantum groups [4], Kac algebras, Hopf algebras, etc. The fundamental idea of all these approaches consists in using the category of linear maps as a basic language. One formulates the notion of a group in this language and then one observes that there is room for "groups with noncommutative space." In the sequel we propose to call such objects "quantum pseudogroups" in order to emphasize the existence of approaches based on categories other than linear (=quantum).

In this paper we study one such approach based on binary relations. Since binary relations are union maps (Sect. 1), the category they form is said to be the union category and we obtain a notion of a union pseudogroup (Sect. 6). A quantization procedure (Sect. 4) applied to each finite union pseudogroup