Commun. Math. Phys. 134, 321–345 (1990)

Communications in
Mathematical
Physics

Critical Limit One-Point Correlations of Monodromy Fields on $\mathbb{Z}^2$

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Received February 1, 1989; in revised form May 19, 1990

Abstract. Monodromy fields on $\mathbb{Z}^2$ are a family of lattice fields in two dimensions which are a natural generalization of the two dimensional Ising field occurring in the C^*-algebra approach to Statistical Mechanics. A criterion for the critical limit one point correlation of the monodromy field $\sigma_a(M)$ at $a \in \mathbb{Z}^2$,

$$\lim_{s \uparrow 1} \langle \sigma_a(M) \rangle ,$$

is deduced for matrices $M \in GL(p, \mathbb{C})$ having non-negative eigenvalues.

Using this criterion non-identity 2×2 matrices are found with finite critical limit one point correlation. The general set of $p \times p$ matrices with finite critical limit one point correlations is also considered and a conjecture for the critical limit n point correlations postulated.

1. Introduction

The C^*-algebra approach to the Ising model via the transfer matrix is now wellknown, see [1, 4, 7–9, 10] for example. Monodromy fields on $\mathbb{Z}^2$, introduced in [14] are a family of lattice fields in two dimensions which are a natural generalization of the two dimensional Ising field. They were inspired by [21] and in a sense are lattice analogues of the continuum fields used in [21, IV] and also in the Federbush and massless Thirring models, see [19, 20, 6]. These lattice fields are interesting for several reasons. Firstly by controlling the scaling limit mathematically precise information on the continuum can be found and this approach was successfully used for the Ising field in [17, 18], secondly there are numerous analogues of continuum structures suggesting a discrete theory on the lattice itself. For $M \in GL(p, \mathbb{C})$ and $a \in \mathbb{Z}^2$ it is possible to define the monodromy field $\sigma_a(M)$ at a. This is a generalization of the Ising field in the sense that when M is the scalar -1 the vacuum expectation of a product $\sigma_{a_1}(-1) \dots \sigma_{a_n}(-1)$ gives the square of an Ising correlation. The motivation for the name "monodromy field" is the fact that