Commun. Math. Phys. 134, 273–292 (1990)

Communications in
Mathematical
Physics


# Quantum Yang–Mills on the Two-Sphere

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Received December 21, 1989; in revised form March 16, 1990

**Abstract.** We obtain the quantum expectations of gauge-invariant functions of the connection on a principal $G = SU(N)$ bundle over $S^2$. We show that the space $\mathscr{A}/\mathscr{G}_m$ of connections modulo gauge transformations which are the identity at one point is itself a principal bundle over $\Omega G$, based loops in the symmetry group. The fiber in $\mathscr{A}/\mathscr{G}_m$ is an affine linear space. Quantum expectations are iterated path integrals first over this fiber then over $\Omega G$, each with respect to the push-forward to $\mathscr{A}/\mathscr{G}_m$ of the measure $e^{-S(A)}\mathscr{D}A$. $S(A)$ denotes the Yang–Mills action on $\mathscr{A}$. There is a global section of $\mathscr{A}/\mathscr{G}_m$ on which the first integral is a Gaussian. The resulting measure on $\Omega G$ is the conditional Wiener measure. We explicitly compute the expectations of a special class of Wilson loops.

## Introduction

We consider the expectation of a gauge-invariant function $f$ with respect to the formal measure $\int_{\mathscr{A}} e^{-S(A)}\mathscr{D}A$, where $S(A) = \frac{1}{4}\|F_A\|^2$ and $\mathscr{A}$ is the space of all connections on a $G = SU(N)$ (trivial) bundle over $S^2$. This measure pushes forward under the projection $\mathscr{A} \to \mathscr{A}/\mathscr{G}_m$, where $\mathscr{G}_m$ is the space of gauge transformations which are the identity at a given point $m \in S^2$. The push-forward measure formally defines a measure $\mu$ which differs from the natural measure on $\mathscr{A}/\mathscr{G}_m$ by a factor describing how the size of the orbit varies within $\mathscr{A}/\mathscr{G}_m$. The devices of gauge-fixing and Faddeev–Popov ghosts give a presumably well-defined measure on $\mathscr{A}$, whose push-forward agrees with $\mu$. This agreement permits us to compute the expectation of $f$ directly on $\mathscr{A}/\mathscr{G}_m$ with respect to the measure $\mu$.

The space $\mathscr{A}/\mathscr{G}_m$ is homotopic to $\Omega G$, based loops on $G$ as shown in Atiyah and Jones [1] and Singer [2]. Section 2.1 presents the homotopy equivalence via a map $\xi: \mathscr{A}/\mathscr{G}_m \to \Omega G$. In fact, $\xi$ is the projection map of the bundle $\mathscr{A}/\mathscr{G}_m$ over $\Omega G$ with an affine space as the fiber.

Integration on $\mathscr{A}/\mathscr{G}_m$ is integration over the affine fibers followed by integration over $\Omega G$, with respect to the measures $\mu$ induces. We exhibit the measure on each fiber as a Gaussian. Integrating over the fibers defines the push-forward measure