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Gibbs' Functionals on Subshifts

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Abstract. We consider functionals on one dimensional subshifts which have prescribed Randon–Nikodym derivative under transportation by conjugating homeomorphisms, and investigate their relation to Ruelle's transfer operator. In particular we show that two-sided functionals essentially are products of a functional which are supported on stable and unstable leaves. We also prove the meromorphicity of the Fourier transform of correlation functions for Axiom A follows in a more general setting

In this paper we study Gibbs' functionals on one dimensional lattice systems and relate them to the eigenspaces to Ruelle's Perron Frobenius operator. The knowledge we have about its spectrum enables us to give a classification of Gibbs' functionals in terms of eigenfunctionals up to a remainder which corresponds to the essential spectrum and therefore remains inaccessible to the technique employed here. However as the essential spectrum is an artefact of the Banach space of functions we are working with, it seems conceivable that a more sensible choice of a function space might remove this difficulty. Using interactions, previous work on this subject was done by Ruelle in [12] and [13]. Some of his results will be translated into a setting using an exponentially decreasing potential instead of interactions whereby one has to rely on Sinai's representation of two-sided functions by cohomologous one-sided ones. Similar to the case of a measure, we define Gibbs' functionals by their behaviour when transported by conjugating homeomorphisms. This characterisation of Gibbs' measures was first considered by Capocaccia [7] who also showed that given a point and its image the germs of conjugating homeomorphisms are locally unique. The need of extending this notion to functionals arose in [12] where the correlation function of Axiom A diffeomorphisms was examined and the region of meromorphicity of its Fourier transform determined. It thereby turned out that using Gibbs' functionals, the residues of simple poles acquire an intriguingly simple form. In the case of poles of higher order the expressions of the negative terms in the Laurent expansion become more involving; however the coefficients can again be expressed through Gibbs' functionals.