Commun. Math. Phys. 134, 89–107 (1990)

Communications in Mathematical Physics

Scalings in Circle Maps (I)

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Received August 28, 1989; in revised form April 27, 1990

Abstract. Let f be a "flat spot" circle map with irrational rotation number. Located at the edges of the flat spot are non-flat critical points ($S: x \to Ax^{\nu}$, $\nu \geqq 1$). First, we define scalings associated with the closest returns of the orbit of the critical point. Under the assumption that these scalings go to zero, we prove that the derivative of long iterates of the critical value can be expressed in the scalings. The asymptotic behavior of the derivatives and the scalings can then be calculated. We concentrate on the cases for which one can prove the above assumption. In particular, let one of the singularities be linear. These maps arise for example as the lower bound of the non-decreasing truncations of non-invertible bimodal circle maps. It follows that the derivatives grow at a sub-exponential rate.

I. Introduction

It is, in our opinion, an important problem how for a smooth family of dynamical systems the varying dynamics in the configuration space is reflected in the parameter space. In one dimension (in the context of unimodal maps of the interval and homeomorphisms of the circle) this problem has been studied in great detail in the case renormalization on a compact set of such maps is uniformly hyperbolic. We will refer to this case as being renormalizable. The context in which we propose to investigate this problem is that of a smooth one-parameter family of bimodal maps $\bar{f}_t$ of the circle.

As for the parameter space, the quantity we are interested in is the lower bound $\varrho(t)$ of the rotation interval associated with f_t. It turns out that f_t always has an order-preserving non-wandering set Ω_t with rotation number $\varrho(t)$, such that

$$\bar{f}_t|_{\Omega_t} = f_t|_{\Omega_t}.$$

Here f_t is the lowest bound on the non-decreasing truncations of $\bar{f}_t$ [as described in Veerman (1989)]. This then reduces the setting to a more manageable one as it allows us to study families of monotone maps with a "flat spot." The precise