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Crystal Base for the Basic Representation of $U_q(\widehat{\mathfrak{sl}}(n))$

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Abstract. We show the existence of the crystal base for the basic representation of $U_q(\widehat{\mathfrak{sl}}(n))$ by giving an explicit description in terms of Young diagrams.

0. Introduction

In [5] Kashiwara introduces the notion of crystal base for integrable representations of $U_q(\mathfrak{g})$, where $\mathfrak{g}$ is any symmetrizable Kac–Moody Lie algebra. The crystal base has a simple structure at $q=0$. Let $\{e_i, f_i, t_i^{\pm}\}$ be a set of generators of $U_q(\mathfrak{g})$. Suppose M is an integrable $U_q(\mathfrak{g})$-module. Kashiwara [5] constructs certain operators $\tilde{e}_i, \tilde{f}_i$ acting on M. These operators are obtained by modifying the simple root vectors e_i and f_i. When M is an irreducible highest weight module with highest weight vecctor u, define:

$$L=\sum A\tilde{f}_{i_1}\tilde{f}_{i_2}\cdots\tilde{f}_{i_k}u\subset M \tag{0.1}$$

and

$$B=\{v=\tilde{f}_{i_1}\tilde{f}_{i_2}\cdots\tilde{f}_{i_k}u\in L/qL \mid v\neq 0\}, \tag{0.2}$$

where $A\subset K=\mathbf{Q}(q)$ is the ring of rational functions in q without pole at $q=0$. Kashiwara [5] conjectures that (L,B) satisfies the following crucial properties:

$$\tilde{e}_iL\subset L \quad\text{and}\quad \tilde{f}_iL\subset L, \quad \text{for all } i, \tag{0.3}$$

$$\tilde{e}_iB\subset B\cup\{0\} \quad\text{and}\quad \tilde{f}_iB\subset B\cup\{0\}, \quad \text{for all } i, \tag{0.4}$$

$$u=\tilde{e}_iv \quad \text{if and only if} \quad v=\tilde{f}_iu, \quad \text{for all } i \quad\text{and}\quad u,v\in B. \tag{0.5}$$

He proves his conjecture for $\mathfrak{g}=\mathfrak{sl}(n)$, $\mathfrak{o}(2n+1)$, $\mathfrak{sp}(2n)$ and $\mathfrak{o}(2n)$ and calls (L,B) the crystal base.

In this paper we prove this conjecture for the basic representation of $U_q(\widehat{\mathfrak{sl}}(n))$ with highest weight Λ_0 ($\Lambda_0(t_i^{\pm})=q^{\pm 1}\delta_{i,0}$). We start with the Fock space representation of $U_q(\widehat{\mathfrak{sl}}(n))$ constructed by Hayashi [3]. We identify the Fock space $\mathscr{F}$ with the space spanned by Young diagrams [4]. Then for each i, we decompose $\mathscr{F}$ with respect to $U_q(\mathfrak{sl}(2))_{(i)}$ generated by $\{e_i, f_i, t_i^{\pm}\}$ (see, Theorem 3.1). This leads to the