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Local Quasiequivalence and Adiabatic Vacuum States

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Abstract. The problem of determining the physically relevant states acquires a new dimension in curved spacetime where there is, in general, no natural definition of a vacuum state. It is argued that there is a unique local quasiequivalence class of physically relevant states and it is shown how this class can be specified for the free Klein–Gordon field on a Robertson–Walker spacetime by using the concept of an adiabatic vacuum state. Any two adiabatic vacuum states of order two are locally quasiequivalent.

1. Introduction

In quantum field theory on Minkowski space the vacuum state is of primary importance. It is a state which is invariant under the group of isometries of Minkowski space time, the Poincaré group. Furthermore it is a ground state, i.e. a state of lowest energy in the corresponding vacuum representation. On an arbitrary Lorentzian spacetime there are in general no isometries so that there is no corresponding way of trying to define a "vacuum" state. Much work has been devoted to the problem of developing other criteria leading to a suitable definition of a vacuum state. It seems probable that there is no unique natural state for a quantum field theory on an arbitrary Lorentz spacetime which deserves the epithet "vacuum state" but at best some class of states which might represent local equilibrium states.

In view of this situation, it is worth reexamining the reasons which lead one to ask for such a vacuum state. The original motivation was undoubtedly to be able to define a suitable notion of particle. The notion of particle even in a Minkowski quantum field theory is a complex one with several facets which we do not wish to examine in detail here. For our purposes, it will be enough to adopt what is for practical purposes by far the most important notion of particle, namely that which underlies scattering theory. Roughly speaking, a particle is an excitation of the vacuum which is relatively well localized in space and time and moves on a straight line path. The importance of the concept of particle is that it allows one