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Selberg Supertrace Formula for Super Riemann Surfaces, Analytic Properties of Selberg Super Zeta-Functions and Multiloop Contributions for the Fermionic String

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Abstract. In this paper a complete derivation of the Selberg supertrace formula for super Riemann surfaces and a discussion of the analytic properties of the Selberg super zeta-functions is presented. The Selberg supertrace formula is based on Laplace-Dirac operators $\square_m$ of weight m on super Riemann surfaces. The trace formula for all $m \in \mathbf{Z}$ is derived and it is shown that one must discriminate between even and odd m. Particularly the term in the trace formula proportional to the identity transformation is sensitive to this discrimination. The analytic properties of the two Selberg super zeta-functions are discussed in detail, first with, and the second without consideration of the spin structure. We find for the Selberg super zeta-functions similarities as well as differences in comparison to the ordinary Selberg zeta-function. Also functional equations for the two Selberg super zeta-functions are derived. The results are applied to discuss the spectrum of the Laplace-Dirac operators and to calculate their determinants. For the spectrum it is found that the nontrivial Eigenvalues are the same for $\square_m$ and $\square_0$ up to a constant depending on m, which is analogous to the bosonic case. The analytic properties of the determinants can be deduced from the analytic properties of the Selberg super zeta-functions, and it is shown that they are well-defined. Special cases ($m = 0, 2$) for the determinants are important in the Polyakov approach for the fermionic string. With these results it is deduced that the fermionic string integrand of the Polyakov functional integral is well-defined.

I. Introduction

The Selberg trace formula has turned out to be a powerful tool to analyse the spectra of Laplacians on Riemann surfaces and to calculate their determinants. In

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