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On the Diffusive Nature of Entropy Flow in Infinite Systems: Remarks to a Paper by Guo-Papanicolau-Varadhan*

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Abstract. The hydrodynamic behaviour of interacting diffusion processes is investigated by means of entropy (free energy) arguments. The methods of [13] are simplified and extended to infinite systems including a case of anharmonic oscillators in a degenerate thermal noise. Following [14, 15] and [3–5] we derive a priori bounds for the rate of entropy production in finite volumes as the size of the whole system is infinitely extended. The flow of entropy through the boundary is controlled in much the same way as energy flow in diffusive systems [4].

0. Introduction

In a recent paper Guo-Papanicolau-Varadhan [13] proposed a new, fairly general approach to the hydrodynamic description of microscopically reversible spin systems in finite volumes. Using the free energy (relative entropy) of the model as a Liapunov function, they found that space-time averages of the evolved state approach a canonical local equilibrium, cf. Holley [14]. Although the parameter of this canonical state, that is the mean spin, has not been identified yet at this stage, a beautiful second entropy argument shows that the mean spin happens to be stable at the macroscopic level, therefore it is controlled by the conservation law. This means that the evolution equation of this conserved quantity closes up in the hydrodynamic limit, and a non-linear diffusion equation is obtained. From a probabilistic point of view, this result is a sophisticated law of large numbers formulated in a functional space; a more advanced technology yields also the related theory of large deviations [2]. The main purpose of this paper is to extend the entropy arguments of [13] to infinite systems, see Fritz [6, 7] and Funaki [10] for some previous results based on a different method. We are interested also in the

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