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Modular Invariants for Affine $\widehat{SU}(3)$ Theories at Prime Heights

Ph. Ruelle[1]*, E. Thiran[1], and J. Weyers[2]*****

[1] Institut de Physique Théorique, Université Catholique de Louvain, B-1348 Louvain-La-Neuve, Belgium
[2] Theory Division, CERN, Geneva 23, Switzerland

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Abstract. A proof is given for the existence of two and only two modular invariant partition functions in affine $\widehat{SU}(3)_k$ theories at heights $n=k+3$ which are prime numbers. Arithmetic properties of the ring of algebraic integers $\mathbb{Z}(\omega)$ which is related to $SU(3)$ weights are extensively used.

1. Introduction

The classification of all modular invariant partition functions of a rational conformal field theory is obviously an important problem. In the case of affine theories, a complete answer is known, at present, only for $\widehat{SU}(2)$ at all levels [1] and for $\widehat{SU}(n)$ at level one [2]. For $\widehat{SU}(3)$ theories two modular invariants have been constructed at all levels [3] and, for exceptional cases, additional invariants are known as well [4]. However, as far as we know, there is no proof, at any level except $k=1$, that these invariants actually exhaust all possibilities.

In this paper we take a little step towards setting up the complete classification of modular invariants for $\widehat{SU}(3)_k$: we will prove that at prime heights $n=k+3$ there are indeed two and only two modular invariant partition functions. The proof makes extensive use of arithmetic properties of the (quadratic) ring of algebraic integers $\mathbb{Z}(\omega)$ which is naturally related to $SU(3)$ weights.

In an affine $\widehat{SU}(3)$ theory, the Hilbert space splits into two chiral parts, each of which decomposes into a finite sum of subspaces corresponding to integrable

* Chercheur IISN, Address after October 1: Dublin Institute for Advanced Studies, Dublin, Ireland
** Chercheur IISN
*** On leave of absence from Institut de Physique Théorique, B-1348 Louvain-La-Neuve, Belgium