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Dimension of the Commutant for the $SU(N)$ Affine Algebras

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Abstract. Explicit formulae are obtained, giving the number of independent matrices which commute with the matrices S and T describing the modular transformations of the $SU(N)$ affine characters.

I. Introduction

In the context of rational conformal field theories, the construction and the classification of modular invariants remains one of the major problems. In a statistical mechanics language, this corresponds to the classification of all fixed-points of the renormalization group in two dimensions.

As the Wess-Zumino-Witten models are thought to be the building blocks in the construction of RCFT's, much attention has been focused on their modular invariants. But although many such invariants are known [1], there is so far no exhaustive list, except in the cases $SU(2)$ [2] and $SU(N)$ at level 1 [3].

Affine modular invariants are sesquilinear forms in the affine characters

$$Z(\tau, \tau^*) = \Sigma\, [\chi_\lambda(\tau)]^*\, N_{\lambda\lambda'}\, [\chi_{\lambda'}(\tau)] \ ,$$

where the coefficients $N_{\lambda\lambda'}$ are subject to appropriate conditions to make $Z(\tau, \tau^*)$ a partition function [2].

For the affine $SU(N)$ algebras, a systematic approach has been initiated by Bauer and Itzykson [4]. They have given a description of the commutant of the (extended) modular transformations carried by the characters. Indeed the modular invariance of $Z(\tau, \tau^*)$ requires that the matrix $N_{\lambda\lambda'}$ belongs to this commutant.

Within the strategy adopted in [2] which led to the ADE classification for the $SU(2)$ invariants, finding the commutant is the first step in the classification program. An interesting alternative is the study of $SU(N)$ lattice integrable models

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