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Special Geometry

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Abstract. A *special manifold* is an allowed target manifold for the vector multiplets of $D=4$, $N=2$ supergravity. These manifolds are of interest for string theory because the moduli spaces of Calabi–Yau threefolds and $c=9$, (2, 2) conformal field theories are special. Previous work has given a local, coordinate-dependent characterization of special geometry. A global description of special geometries is given herein, and their properties are studied. A special manifold $\mathscr{M}$ of complex dimension n is characterized by the existence of a holomorphic $Sp(2n+2, R) \otimes GL(1, C)$ vector bundle over $\mathscr{M}$ with a nowhere-vanishing holomorphic section Ω. The Kähler potential on $\mathscr{M}$ is the logarithm of the $Sp(2n+2, R)$ invariant norm of Ω.

I. Introduction

The construction of a supersymmetric field theory proceeds by demanding that the action is invariant under some chosen group of supersymmetry transformations. This places constraints on the particle content and couplings of the theory. In theories with scalars, by viewing the scalar fields as coordinates on a target manifold $\mathscr{M}$, it is often possible to reinterpret these constraints as constraints on the geometry of $\mathscr{M}$. This reinterpretation is not always straightforward, because the constraints arising from supersymmetry are expressed locally and in a particular coordinate system on $\mathscr{M}$.

As examples, it is known that local $N=1$, supersymmetry in four dimensions requires that $\mathscr{M}$ is a Kähler manifold [1] of restricted type[1] [2]. Local $N=2$ supersymmetry with $(0, \frac{1}{2})$ chiral multiplets requires that $\mathscr{M}$ is quaternionic [3].

Oddly enough, the allowed geometry of the target space $\mathscr{M}$ of locally $N=2$ supersymmetric $(0, \frac{1}{2}, 1)$ vector multiplets [4] – we shall refer to this as *special*

[1] This means that the Kähler form $\mathscr{J}$ is an even element of integral cohomology, or equivalently that there exist a line bundle with first Chern class equal to $[\mathscr{J}]$