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Log-Sobolev Inequalities for Infinite One Dimensional Lattice Systems

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Abstract. It is shown that a unique Gibbs measure of infinite spin system with short range interaction on one dimensional lattice satisfies log-Sobolev inequality.

0. Introduction

Log-Sobolev inequalities (shortly log-S) have been introduced in [1] and since then many investigations of them and related problems have been published. (For a recent bibliographical review see [2].) Although one of the important features of these inequalities is the fact that they generalize the classical Sobolev inequalities to infinite dimensional spaces, there are only few papers dealing specifically with the infinite dimensional case. Let us shortly describe them. In [1] it has been shown that any infinite product of probability measures $\{\rho_n\}_{n\in\mathbb{N}}$ satisfies log-S inequality with a coefficient $0 < c < \infty$, provided each measure ρ_n satisfies log-S with a corresponding coefficient $0 < c_n < c$.

Moreover, using this fact, in the same paper it has been proven that also any Gaussian measure satisfies log-S. This, together with a general theory developed in [1], yields an elegant proof of hypercontractivity estimates of Nelson [3] (see also [4]) for the free field, so important in development of euclidean field theory.

The first example of probability measures on an infinite dimensional space satisfying log-Sobolev inequalities and not being of product or Gaussian type appeared in [5]. The authors used the Γ_2-criterion of Bakry and Emery [6] to prove these inequalities for the measures of classical statistical mechanical systems on a lattice with single spin space given by the S^d-sphere $d \geqq 2$ and at sufficiently high temperatures. The authors of this paper, Carlen and Stroock, were motivated by investigation of Markov semigroups and in particular by applications to the study of stochastic dynamics in statistical mechanical systems ([7–12]).

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