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Relating the Approaches to Quantised Algebras and Quantum Groups

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Abstract. This paper constructs two representations of the quantum group $U_q g'$ by exploiting its quotient structure and the quantum double construction. Here the quantum group is taken as the dual to the quantised algebra $U_q g$, a one parameter deformation of the universal enveloping algebra of the Lie algebra g, as in Drinfel'd [6] and Jimbo [10]. From the two representations, the Hopf structure of the quantised algebra $U_q g$ is reexpressed in a matrix format. This is the very structure given by Faddeev et al. [7], in their approach to defining quantum groups and quantised algebras via the quantisation of the function space of the associated Lie group to g.

Introduction

A newcomer to the field of quantum groups will encounter four essential papers on the structure and definition of quantised algebras and quantum groups, namely those by Jimbo [9, 10], Drinfel'd [6] and Faddeev et al. [7]. These works define the concepts of quantised algebras and quantum groups using two alternative approaches. The first two authors use a more mathematical formulation for defining a quantised algebra, introducing a one parameter deformation of an universal enveloping algebra of a Lie (or Kac Moody) algebra. The concepts are rather intricate, and for this reason the approach of Faddeev et al. [7]—based on a quantisation of the function space of the accompanying Lie group—may well be more appealing initially, especially to the physics community. However the two approaches remain rather disjoint, the connection between the two being elusive, the reader only having claims of their equivalence in [7].

As discussed in Drinfel'd [6], the motivation for introducing the one parameter deformation of the universal enveloping algebra comes from the classical isomorphism between the function space of the (connected) Lie group G and the

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