Commun. Math. Phys. 133, 53–74 (1990)

Communications in Mathematical Physics

Existence of Standing Waves for Dirac Fields with Singular Nonlinearities

Mikhael Balabane[1,2], Thierry Cazenave[3] and Luis Vázquez[4]

[1] Département de Mathématiques, Université de Reims-Champagne-Ardennes, B.P. 347, F-51062 Reims Cedex, France
[2] CMA, École Normale Supérieure, 45, rue d'Ulm, F-75252 Paris Cedex 05, France
[3] Analyse Numérique, Université Pierre et Marie Curie, 4, place Jussieu, F-75252 Paris Cedex 05, France
[4] Departamento de Fisica Teorica, Facultad de Ciencias Fisicas, Universidad Complutense, E-28040 Madrid, Spain

Received July 10, 1989; in revised form February 2, 1990

Abstract. We prove the existence of stationary states for nonlinear Dirac equations of the form

$$i \sum_{\mu=0}^{3} \gamma^\mu \partial_\mu \psi - M\psi + F(\bar{\psi}\psi)\psi = 0, \tag{E}$$

where $M > 0$ and F is a singular self-interaction. In particular, in the model case where $F(s) = -s^{-\alpha}$, for some $0 < \alpha < 1$, and for every $\omega > M$, there exists a solution of (E) of the form $\psi(t,x) = e^{i\omega t}\varphi(x)$, where $x_0 = t$ and $x = (x_1, x_2, x_3)$, such that φ has compact support. If $0 < \alpha < 1/3$, then φ is of class C^1. If $1/3 < \alpha < 1$, then φ is continuously differentiable, except on some sphere $\{|x| = R\}$, where $|\nabla\varphi|$ is infinite.

1. Introduction

In this paper, we study the existence of stationary states for nonlinear Dirac equations of the form

$$i \sum_{\mu=0}^{3} \gamma^\mu \partial_\mu \psi - M\psi + F(\bar{\psi}\psi)\psi = 0. \tag{1.1}$$

We consider here the case where F is a **singular** self-interaction.

The notation is the following. $\psi: \mathbf{R}^4 \to \mathbf{C}^4$, $\partial_\mu = \partial/\partial x_\mu$, M is a positive constant, $\bar{\psi}\psi = (\gamma^0\psi, \psi)$, where $(\cdot,\cdot)$ is the usual scalar product in $\mathbf{C}^4$ and the γ^μ's are the 4×4 matrices of the Pauli–Dirac representation (see [14, 15, 17, 18]), given by

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \quad \text{and} \quad \gamma^\mu = \begin{pmatrix} 0 & \sigma^k \\ -\sigma^k & 0 \end{pmatrix}, \quad \text{for} \quad k = 1, 2, 3,$$