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Asymptotic Solutions of the Elastic Wave Equation and Reflected Waves near Boundaries

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Abstract. In the first half of this paper, we construct asymptotic solutions of linear anisotropic elastic equations. In the latter half, we investigate waves reflected by boundaries for plane incident waves in terms of these solutions. Especially, it is examined whether or not the mode-conversion occurs near points where the incident waves hit the boundaries perpendicularly.

Introduction

Let Ω be a domain in $\mathbb{R}^n_x(x = {}^t(x_1, \ldots, x_n), n \geqq 2)$ with a C^∞ boundary $\partial\Omega$, and consider the elastic wave equation

$$\left(\partial_t^2 - \sum_{i,j=1}^{n} a_{ij}\partial_{x_i}\partial_{x_j}\right)u(t,x) = 0 \quad \text{in} \quad \mathbb{R} \times \Omega.$$

Here, $u = {}^t(u_1, \ldots, u_n)$ is the displacement vector, and a_{ij} are real constant $n \times n$-matrices whose (p,q)-components are denoted by a_{ipjq}. We assume that a_{ij} satisfy

$$a_{ipjq} = a_{pijq} = a_{jqip}, \quad i, j, p, q = 1, 2, \ldots, n, \tag{A.1}$$

$$\sum_{i,p,j,q=1}^{n} a_{ipjq}\varepsilon_{jq}\bar{\varepsilon}_{ip} \geqq \delta \sum_{i,p=1}^{n} |\varepsilon_{ip}|^2 \quad \text{for every Hermitian matrices } (\varepsilon_{ij}), \tag{A.2}$$

$$\sum_{i,j=1}^{n} a_{ij}\xi_i\xi_j \text{ has eigenvalues of constant multiplicity for}$$
$$\text{any } \xi = {}^t(\xi_1, \ldots, \xi_n) \in \mathbb{R}^n - \{0\}. \tag{A.3}$$

In the isotropic case (i.e. $a_{ipjq} = \mu(\delta_{pq}\delta_{ij} + \delta_{iq}\delta_{jp}) + \lambda\delta_{ip}\delta_{jq}, \lambda + \frac{2}{n}\mu > 0$ and $\mu > 0$), the above assumptions are all satisfied. Let $\{\lambda_l(\xi)\}_{l=1,\ldots,d}$ be the distinct eigenvalues of $\sum_{i,j=1}^{n} a_{ij}\xi_i\xi_j$. Then, $\lambda_l(\xi)$ become positive C^∞ functions ($\xi \neq 0$). We denote by $P_l(\xi)$ the projection into the eigenspace of $\lambda_l(\xi)$.