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## Extended Super-Kač–Moody Algebras and Their Super-Derivation Algebras

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Abstract. We study the N-extended super-Kač-Moody algebras, i.e. extensions of the Lie algebra of the loop group over the super-circle  $A_N$ . The extensions are characterized by 2-cocycles which are computed in terms of the cyclic cohomology of the Grassmann algebra with N generators. The graded algebra of super-derivations compatible with each extension is determined. The cases N = 1, 2, 3 are examined in detail and their relation with the Ademollo et al. superconformal algebras is discussed. We examine the possibility of defining new superconformal algebras which, for N > 1, generalize the N = 1 Ramond-Neveu-Schwarz algebra.

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