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Random Fields as Solutions of the Inhomogeneous Quaternionic Cauchy-Riemann Equation. I. Invariance and Analytic Continuation

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Abstract. We consider random fields A satisfying the quaternionic Cauchy-Riemann equation $\partial A = F$, where F is white noise. Under appropriate conditions on F, A is invariant under the proper Euclidean group in four dimensions, but in general not under time reflection. The Schwinger functions can be analytically continued to Wightman functions satisfying the relativistic postulates on invariance, spectral property and locality.

1. Introduction

In recent years models of local interacting relativistic quantum fields of scalar, vector, or gauge type have been constructed (see e.g. [2, 6, 7, 20, 28, 34]) in space-time dimensions $d<4$. Since the basic work of Nelson's, the construction of such fields has been closely connected with the construction of Euclidean (i.e. invariant in law under the Euclidean group) random fields over $\mathbf{R}^d$ having suitable "Markovian" properties.[1] The case $d=4$ however, has remained open, from the quantum field as well as the Euclidean Markov field point of view.[2]

The present article is the first one in a planned series where we investigate random fields and quantum fields over $\mathbf{R}^4$. Our 4-component random fields A are obtained by solving a first order elliptic partial differential equation

$$\partial A = F, \tag{1.1}$$

where F is a 4-component Gauss-Poisson white noise with specific Euclidean invariance. We shall formulate the above by making use of ample algebraic structure of the field $\mathbf{H}$ of quaternions, which is isomorphic to $\mathbf{R}^4$ as vector spaces over the reals.

Some results have already been announced in [0] and [3–5]. Reference [12] contains a correction to [5] as well as further discussions and proofs.

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[1] For $d=2$ most scalar models have been shown to fulfill the (strict) global Markov property (in particular with respect to half-spaces) and Nelson's axioms. See [13] and references therein

[2] For some partial results, see e.g. [0, 1, 6–9, 14, 18] and references therein