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Isoperimetric Phase Transitions of Two-Dimensional Droplets

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Abstract. We consider two-dimensional assemblies of particles governed by hamiltonians depending on the area and the perimeter of their convex hull. Provided the hamiltonian is quadratically homogeneous in the coordinates, we find an exact formula for the free energy. Phase transitions resulting from the competition between area and perimeter can easily be produced and explicitly dealt with. We illustrate those features by a simple example undergoing a second-order transition.

1. Introduction

The competition between the area and the perimeter constitutes, as attested, e.g. by Queen Dido's problem or isoperimetric and Bonnesen inequalities [1, 2], one of the central themes in the development of two-dimensional geometry. To be able to exploit this competition in statistical mechanics, we have to find a way to:

i) generate an ensemble of shapes together with an a priori distribution maximizing the entropy
ii) assign an energy to each individual shape.

In two recent publications [3, 4], we have started a systematic investigation of the statistical mechanical behaviour of a two-dimensional assembly of particles, whose interaction potential depends *on the convex hull spanned by the particles only.* One can think of this approach as an attempt to derive from a purely microscopic basis the distribution function of a fluctuating container in a pressure ensemble. Put in another way, such models seek to describe a drop in equilibrium with a mechanical reservoir in terms of its $2N \approx 10^{23}$ microscopic degrees of freedom. As done in [3, 4], we concentrate on interactions of global geometric type only, the two-body interactions between the particles being discarded.

Our approach differs from mainstream statistical mechanical treatment of interfaces (see e.g. [5–9] and references therein) where *macroscopic* interfaces