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Constrained Random Walks and Vortex Filaments in Turbulence Theory*

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Abstract. We consider a simplified model of vorticity configurations in the inertial range of turbulent flow, in which vortex filaments are viewed as random walks in thermal equilibrium subjected to the constraints of helicity and energy conservation. The model is simple enough so that its properties can be investigated by a relatively straightforward Monte-Carlo method: a pivot algorithm with Metropolis weighting. Reasonable values are obtained for the intermittency dimension D, a Kolmogorov-like exponent γ, and higher moments of the velocity derivatives. Qualitative conclusions are drawn regarding the origin of non-gaussian velocity statistics and regarding analogies with polymers and with systems near a critical point.

Introduction

Three dimensional incompressible flow can be approximated by following the evolution of a collection of vortex tubes, and discretizations that consider finite collections of tubes lead to vortex approximations [1, 4, 7, 18, 21]. One can consider the finite approximations as models of the Navier–Stokes equations and examine their statistical properties to the hope of gaining an understanding of turbulence. One must of course be aware that the properties of finite systems do not necessarily survive the passage to the limit of a continuous system. In particular, the finite systems greatly simplify the geometric complexity of the microstructures that occur in real turbulence.

The inertial range of scales in turbulent motion is the range of scales intermediate between the scales on which the fluid is stirred and the scales on which its energy is dissipated. These scales play a key role in the dynamics of turbulence. In the inertial range, turbulent flow can be viewed as being in approximate thermal equilibrium. If one represents the flow on these scales by a collection of vortex tubes, one can appeal to methods of analysis adapted from other branches of

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