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General Zakharov–Shabat Equations, Multi-Time Hamiltonian Formalism, and Constants of Motion

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Abstract. We construct a Hamiltonian formalism for general Zakharov–Shabat equations (zero curvature equations with rational dependence on a parameter) as well as their constants of motion, and prove that the latter are in involution. The field-theoretical (multi-time) Hamiltonian formalism is used.

Introduction

As soon as the complete integrability of the KdV equation was discovered it was also found that this equation was a Hamiltonian system. Later on Hamiltonian structures were suggested for all hierarchies of integrable equations: for generalized KdV, for KP, AKNS etc. However this was not done for the most general scheme for obtaining integrable systems given by Zakharov and Shabat. They considered a zero curvature equation with a rational dependence on a spectral parameter.

Here we construct a Hamiltonian structure for these equations. We have a typical case when a field-theoretical (or multi-time) formalism is natural since both independent variables are equal by right and there is no reason to prefer one of them to the other as a time variable in which the system evolves. It is enough to know a Lagrangian, and then all the elements of the formalism are recovered automatically. Our construction of the Lagrangian generalizes that given by Zakharov and Mikhailov [8] for the case of simple poles.

The Lagrange–Hamiltonian field-theoretical formalism is well-known. However, the specific form we use fits very well to integrable systems under consideration. It has a formal-algebraic character and admits easy calculations. Briefly it was described in a lecture [11] (an earlier sketch in [4]). A complete description is in [3].

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