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# Space-Time Fields and Exchange Fields

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**Abstract.** We derive discrete symmetries of braid group statistics related to charge conjugation and outer automorphisms of the local algebra. The structure of the latter (which are abelian superselection charges) is analyzed in some detail. We use the results to study in great generality a phenomenon recently observed in conformal quantum field theories: the existence of two-dimensional space-time fields with conventional (local, fermionic, dual) commutation relations, expressible as bilinear sums over light-cone fields with exchange algebra commutation relations.

## 1. Introduction

Braid group statistics is the natural statistics in low space-time dimensions. A fundamental reason why it has for a long time escaped our attention, except for a class of "anyon" models with abelian representations of the braid group, is the intimate relation between statistics and internal symmetries: the ordinary (Lie group) symmetries go along with permutation group statistics only [1], while braid group statistics signals a new type of "quantum symmetry" [2] which we are only recently beginning to understand.

While braid group statistics must be expected in up to $2+1$ space-time dimensions (at least for "gauge charges" localized in narrow tubes extending to space-like infinity), the only explicit occurrence of non-abelian braid group statistics so far is in $1+1$ dimensional conformal quantum field theories. Even there, it is not the statistics associated with the local two-dimensional observables that has been read off the Wightman functions (conformal block functions) completely determined by Ward identities, but in fact the statistics of its "chiral" local light-cone fields, i.e. an effectively one-dimensional quantum field theory with particularly simple kinematics.

By virtue of conformal covariance, space-time fields factorize as bilinear expressions in light-cone fields. The monodromy (analyticity) properties of conformal block functions [3], interpreted as vacuum expectation values of light-