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Quantum-Mechanical Calculations in the Algebraic Group Theory

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Abstract. Quantum oscillators on simple Lie algebras satisfying the special symmetry conditions are considered. Statsums, the Witten index and some simple correlators are calculated. The relations between these expressions and orders of algebraic groups over finite fields $\mathbb{F}q$ and degrees of some their representations are established under the condition that the temperature T of systems is equal to $T=\omega/\ln q$. We consider the conformal limit of the theories where ranks of groups go to infinity. Also we discuss the relation between the adelic limit of the theories and the Tamagawa numbers.

1. Introduction

In the last several years methods of the quantum field theory were successfully applied in different domains of mathematics [1]. As a rule they allow to obtain the independent and transparent proofs of mathematical theorems. The aim of the present article is more modest – we give here only the quantum-mechanical interpretation of well-known formulae in the algebraic group theory. Nevertheless we hope that this approach leads to a deeper understanding of some relations in this subject.

The first part of the formulae concerns the orders of finite simple groups. The classification of these groups had been completed recently (see, for example [2]). The grand theorem states that the list of groups contains besides the alternating groups and groups of Lie type twenty six sporadic groups. In the present article we consider only the groups of Lie type over finite fields (GLT). Namely we establish the connection between the orders of GLT and statsums of quantum oscillators on simple Lie algebras over $\mathbf{R}$. More precisely statsums are calculated only for the part of GLT – the Chevalley groups. For the last groups – the so-called twisted groups – it is necessary to calculate the quantum average of twist operators. The connection appears when the temperature T is equal to $\omega/\ln q$, where ω is a frequency of oscillators, $q=p^r$ and p is a prime number. The similar situation