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Quantum Groups and Generalized Statistics in Integrable Models

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Abstract. The paper deals with the integrable massive models of quantum field theory. It is shown that generalized statistics of physical particles is closely connected with the invariance under quantum groups. This invariance provides the possibility to construct quasi-local operators (parafermions) possessing generalized statistics which interpolates the physical particles. For the particular examples of SG, RSG models and scaling 3-state Potts model the parafermions are described completely (all their matrix elements in the space of states are presented).

Introduction

In two-dimensional space-time the quantum fields can satisfy the commutation relations which generalize the usual bosonic and fermionic ones [1, 2]. The most general form of these relations is

$$O_i(x)\,O_j(y) = O_\ell(y)\,O_k(x)\,R_{ij}^{k\ell}\,, \qquad x<y \tag{0.1}$$

where x, y are space coordinates. The matrix R should be a constant solution of the Yang-Baxter equation:

$$R_{12}R_{13}R_{23} = R_{23}R_{13}R_{12}\,.$$

Here we used usual tensor notations (R_{ij} acts nontrivially in the tensor product of i^{th} and j^{th} spaces). Even diagonal matrices R are of interest in this context; they correspond, for example, to kinks in the Gross-Neveu Model [2] or parafermions in Z_N models [3]. The commutation relations (0.1) with nontrivial R-matrices appeared originally in the framework of conformal field theory [4–6].

In the present paper we show how to realize the relations (0.1) for massive completely integrable models and explain the connection between generalized statistics and symmetries associated with quantum groups. The S-matrix in the completely integrable model is factorizable. The two-particle S-matrix should