Commun. Math. Phys. 132, 285–292 (1990)

Communications in Mathematical Physics

On Listing's Law

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Dedicated to Res Jost and Arthur Wightman

Abstract. Listing's law states that visual directions of sight are related to rotations of the eye so that all rotation axes lie in a plane. The geometry of $SO(3)$ indicates several plausible algorithms how the human brain could relate vision to eye movements satisfying Listing's law, and suggests crucial experiments which we have carried out.

1. Introduction

Mathematical objects, like the quantum mechanical rotation group $SU(2)$, are intrinsically beautiful and "unreasonably efficient" in theories of our physical world. Both R. Jost and A.S. Wightman have made wonderful use of $SU(2)$ in their scientific work and teaching. Mathematicians often haven the Platonic attitude that objects like $SU(2)$ are ideas which exist independently of our material world. Neurobiologists see them more materialistically related to cortical activity patterns in the brains of ensembles of mathematicians and laid down in their published common knowledge. While the philosophical issue [1] is undecidable, a natural scientist can ask the more modest question, whether the $SU(2)$ is represented in our brain and, if so, whether in a given hardware implementation of the nervous system, as coarsely described by neuro-anatomy, geometry allows us to make experimentally refutable predictions about the operations of the human neural network.

The $SO(3)$ is intrinsically linked to our sensori-motor periphery. In the labyrinth of the inner ear, for instance, rotatory movements of the head induce flow patterns in 3 almost orthogonal semicircular canals, which at intermediate frequencies are transduced as head angular velocity. The $SO(3)$ also appears in the configuration space of our limbs and, in its purest form, in the kinematics of the eye.

The purpose of this note is to explain some simple geometrical ideas around Listing's law for eye movements and their relation to foveal vision and central visuo-oculomotor transformations in man. The geometry of $SU(2)$ suggests novel ways for interpreting neural activity in the brain in terms of parameters of the external