Commun. Math. Phys. 132, 177–188 (1990)

Communications in
Mathematical
Physics

Resonances, Metastable States and Exponential Decay Laws in Perturbation Theory

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Dedicated to Res Jost and Arthur Wightman

Abstract. Resonances which appear as perturbed bound states are discussed in the framework of Balslev-Combes theory. The corresponding metastable states are constructed using the formal perturbation expansion to order $N-1$ for the (nonexistent) perturbed bound states. They are shown to have exponential decay in time governed by the complex resonance energies, up to a background of order $2N$ in the perturbation parameter. The results apply in lowest order $N=1$ to the perturbation of bound states embedded in the continuum and in arbitrary order to cases like the Stark effect.

1. Introduction

According to standard textbook wisdom, resonances of quantum systems correspond to metastable states which show exponential decay. The difficulties in making this statement precise are notorious and well explained in [13]. While Gamov's one-particle α-decay model still receives attention (see e.g. [16, 17]), relatively little is known in more general situations. In many cases the Balslev-Combes theory of dilation analytic systems [1, 11] or one of its variants [3, 7, 14, 15] allows an elegant definition of the complex resonance energies, but there is yet no general description of the corresponding metastable states and their time evolution. Notable progress has recently been made by Orth [10], who developed a theory of resonances for N-body Schrödinger operators based on the Mourre estimate rather than dilation analyticity. The scope of our present contribution is more limited. We will use dilation analyticity and perturbation theory in the spirit of Simon [12] to discuss resonances which appear as perturbed bound states. By the same approach we can cover cases like the N-body Stark effect, where some results on exponential decay have previously been given by Herbst [4].

In the Balslev-Combes theory, the resonance energies are the complex eigenvalues of a "dilated" Hamiltonian $H(\Theta)$. For a typical N-body system, $H(\Theta)$ has a spectrum of the form shown in Fig. 1. To understand exponential decay in this setting we must answer the following questions. What is – in good approximation –