Commun. Math. Phys. 132, 155–176 (1990)

Communications in Mathematical Physics

Master Symmetries of the XY Model

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Dedicated to Res Jost and Arthur Wightman

Abstract. Master symmetries, found by Barouch and Fuchssteiner for a finite size XY model with the help of a computer program, are mathematically analyzed for an infinitely extended XY model by a rigorous operator algebraic method with an easy computation. The infinite family of commuting Hamiltonians and the master symmetries generating them form an infinite dimensional Lie group of automorphisms of a C^*-algebra of observables for the model.

1. Introduction

The one-dimensional XY-model in statistical mechanics of the spin 1/2 lattice system is known to be exactly solvable. One possible feature of an exactly solvable quantum model is the existence of a commuting family of explicitly describable operators (constants of motion or symmetry generators) which commute with the Hamiltonian of the model. Barouch and Fuchssteiner [7] found an interesting mechanism of creating such a commuting family, which will be quoted in detail in the next section.

Barouch and Fuchssteiner refers the proof to a computer computation. They give only the first few operators in the commuting family explicitly and no general explicit forms for the operators in an infinite family are given. The complicated expressions for the first few operators do not seem to suggest any general explicit form either. Thus we are not sure about the proof of the claim for the general operators in the infinite family, e.g. the proof of their commutativity.

Also Barouch and Fuchssteiner do not specify the boundary condition for the model. The expression for the master symmetry containing the number j of the lattice site explicitly excludes the possibility of the periodic boundary condition. However the validity of the commutativity can be broken at the boundary for other boundary conditions.

* Supported in part by Mombusho International Scientific Research Program