# Construction of Convergent Simplicial Approximations of Quantum Fields on Riemannian Manifolds

Sergio Albeverio[*][**] and Boguslav Zegarlinski

Fakultät für Mathematik, Ruhr-Universität D-4630 Bochum, Federal, Republic of Germany and SFB 237-Bochum-Essen-Düsseldorf

*Dedicated to Res Jost and Arthur Wightman*

**Abstract.** We construct simplicial approximations of random fields on Riemannian manifolds of dimension $d$. We prove convergence of the fields to the continuum limit, for arbitrary $d$ in the Gaussian case and for $d = 2$ in the non-Gaussian case. In particular we obtain convergence of the simplicial approximation to the continuum limit for quantum fields on Riemannian manifolds with exponential interaction.

## 0. Introduction

Quantum fields on Riemannian manifolds have become recently a topic of major interest in several connections. Besides the well known open problem of formulating a mathematical theory of quantum gravity, in which quantum fields on manifolds are traditionally thought to play a fundamental role, see e.g. [As], [HaI], we might mention the explosion of activity in the study of quantum strings (and superstrings), in which conformal fields over two dimensional Riemannian manifolds play a central role, see e.g. [GSW], [AHKPS1,2]. More generally the study of conformal fields with their relations to representation theory of infinite dimensional algebras and groups (see e.g. [Kac], [PrS], [FreLM], [AHKMTT]), statistical mechanics of 2-dimensional lattice systems (see e.g. [Kau], [ISZ]), completely integrable systems (see e.g. [ISZ]) and topological objects like knots and braids (see e.g. [ISZ], [Frö], [RehS], [Ga]) has attracted in recent years great interest, both in mathematics and physics. We also mention that the theory of geometrical fields like gauge fields, Markov cosurfaces, Higgs fields (see e.g. [BaJ], [Gr2], [GrKS], [AHKH], [AHKHK1,2,3], [AHKI], [AIK]) also can be looked at as a natural extension of the theory of scalar quantum fields on Riemannian manifolds.

Also from the point of view of stochastic analysis there is interest in studying random processes and fields on Riemannian manifolds. To quote a couple of contexts where this interest arises let us mention the role played by properties of

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[*] BiBoS Research Centre
[**] CERFIM (Locarno)