Commun. Math. Phys. 131, 381–429 (1990)

Communications in
Mathematical
Physics

Mathematical Models with Exact Renormalization for Turbulent Transport

Marco Avellaneda[1,*] and Andrew J. Majda[2,**]

[1] Courant Institute, New York University, New York, NY 10012, USA

[2] Department of Mathematics and Program in Applied and Computational Mathematics, Princeton University, Princeton, NJ 08544, USA

Abstract. The advection-diffusion of a passive scalar by incompressible velocity fields which admit a statistical description and involve a continuous range of excited spatial and/or temporal scales is very important in applications ranging from fully developed turbulence to the diffusion of tracers in heterogeneous porous media. A variety of renormalization theories which typically utilize partial resummation of divergent perturbation series according to various recipes have been applied to this problem in various contexts. In this paper, a simple model problem for the advection-diffusion of a passive scalar is introduced and the complete renormalization theory is developed with full mathematical rigor. Explicit formulas for the anomalous time scaling in various regimes as well as the Green's function for the large-scale, long-time, ensemble average are developed here. Formulas for the renormalized higher order statistics are also developed. The simple form of the model problem is deceptive; the renormalization theory for this problem exhibits a remarkable range of different renormalization phenomena as parameters in the velocity statistics are varied. These phenomena include the existence of several distinct anomalous scaling regimes as the spectral parameter $\tilde{\varepsilon}$ is varied as well as explicit regimes in $\tilde{\varepsilon}$ where the effective equation for the ensemble average is not a simple diffusion equation but instead involves an explicit random nonlocal eddy diffusivity. We use Fourier analysis and the Feynman-Kac formula as main tools in the explicit exact renormalization theory developed here.

* Research partially supported by NSF DMS8802739, ARO DAAL03-89-K-0039

** Research partially supported by NSF DMS8702864, ARO DAAL03-89-K-0013, ONR N00014-89-J-1044