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Flat Connections and Geometric Quantization

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Abstract. Using the space of holomorphic symmetric tensors on the moduli space of stable bundles over a Riemann surface we construct a projectively flat connection on a vector bundle over Teichmüller space. The fibre of the vector bundle consists of the global sections of a power of the determinant bundle on the moduli space. Both Dolbeault and Čech techniques are used.

0. Introduction

The new invariants of 3-manifolds introduced by Witten [21] can be approached by defining a vector space V canonically associated to a closed surface Σ, a Lie group G, and an integer k. These spaces are to be thought of as analogues of cohomology groups, though satisfying different functorial properties [17]. To define cohomology groups one usually requires a choice of auxiliary structure – a triangulation, Čech covering, differentiable structure, or Riemannian metric – and one needs to prove that the resulting space is independent, in a suitable sense, of that choice. The same is true of the vector spaces required for Witten's theory, and the aim of this paper is to prove that independence for the case $G=SU(m)$.

The underlying idea behind the vector space V is that of the *geometric quantization* of a symplectic manifold M. Given the group G, we consider the space of irreducible representations of the fundamental group $\pi_1(\Sigma)$ into G:

$$\mathrm{Hom}(\pi_1(\Sigma), G)^{\mathrm{irr}}/G,$$

which is in a canonical way a symplectic manifold M. Multiplying the canonical symplectic form by the level k gives it a different symplectic structure. These symplectic manifolds are clearly canonically associated to the surface Σ. To quantize them, in the Kostant-Kirillov-Souriau sense, requires a choice of polarization and one then needs to prove that the space is independent of that

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