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Bound on the Ionization Energy of Large Atoms

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Abstract. We present a simple argument which gives a bound on the ionization energy of large atoms that implies the bound on the excess charge of Fefferman and Seco [2].

1. Introduction

A system consisting of a nucleus of charge Z and N electrons is described by the Schrödinger operator

$$H_{N,Z}=\sum_{i}^{N}\left(-\Delta_i-\frac{Z}{|x_i|}\right)+\sum_{1\leqq i<j\leqq N}\frac{1}{|x_i-x_j|} \tag{1}$$

acting on the antisymmetric space $\mathscr{H}_F=\bigwedge_{i=1}^{N}(L^2(\mathbb{R}^3)\otimes\mathbb{C}^2)$. Here we have assumed for simplicity that the nucleus is infinitely heavy. We call such a system an atom.

The **ground state energy** of the atom is

$$E(N,Z)=\inf\operatorname{spec}_{\mathscr{H}_F}H_{N,Z} \tag{2}$$

and the **ionization energy** is defined as

$$I(N,Z)=E(N-1,Z)-E(N,Z). \tag{3}$$

This is the energy which binds the atom together. It is well known that there is a critical number of electrons $N_c(Z)$ such that

$$I(N_c,Z)>0 \quad \text{and} \quad I(N,Z)=0 \quad \text{if} \quad N>N_c$$

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