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A Unified Approach to String Scattering Amplitudes

A. A. Voronov*

Department of Mathematics, Moscow State University, SU-117234 Moscow, USSR

Abstract. 1) *Physics.* In the calculation of g-loop string tachyon amplitudes with n scattering points the distinguished Polyakov measure $d\pi_{g,n}$ on the moduli space $\mathcal{M}_{g,n}$ of Riemann surfaces of genus g with n punctures arises. We give an interpretatiton of this measure as the modulus squared of a holomorphic section $\mu_{g,n}$ (the Mumford form) of a certain holomorphic line bundle, i.e., we prove an analog of the Belavin-Knizhnik theorem $d\pi_{g,n}=|\mu_{g,n}|^2$ in the amplitudic case. We give an expression for this measure through the determinants of the Laplace operators over ghosts and over multivalued fields with monodromy prescribed by momenta at the scattering points. We show also that the form $\mu_{g,n}$ $(n\geqq 0)$ for the partition function and n-point amplitudes can be obtained from a unified over all n, universal Mumford form.

2) *Mathematics.* The following new concepts from the theory of complex algebraic curves are investigated: divisors with complex coefficients, complex powers of holomorphic line bundles, determinants of Laplace operators over multivalued functions, etc. The corresponding generalizations of the determinant line bundles, the Weil-Deligne pairings, the Quillen and the Arakelov-Deligne metrics are constructed. A suggested by string amplitude considerations analog of the Mumford theorem on holomorphic triviality of the bundle $\lambda_2\otimes\lambda_1^{-13}$ over the moduli space is given. This analog asserts the existence of a canonical flat metric on a certain line bundle $\lambda_2\otimes\lambda_1^{-13}\otimes\left(\bigotimes_{\nu=1}^{13}\langle\mathcal{O}(D^\nu),\mathcal{O}(D^\nu)\rangle^{-1}\right)$ (see the main body of the text). There exist two differences: the latter bundle is not holomorphically trivial but has a canonical flat metric, and, being defined on the Teichmüller space $T_{g,n}$, this bundle can be pulled down only on an infinite-sheeted covering of the moduli space $\mathcal{M}_{g,n}$. The universal isometries and the relative curvatures from the second part of the paper may be interesting, too.

* Address after July 1990: Department of Mathematics, University of California, Davis, CA 95616, USA