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Breit–Wigner Formula for the Scattering Phase in the Stark Effect

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Abstract. Near resonance energy, we study the asymptotic behavior of the derivative of the scattering phase as the applied electric field tends to zero. We obtain the leading asymptotics of the spectral function near a simple resonance, and as an application we rigorously prove the Breit–Wigner formula which relates the width of resonances to the time delay of particles in a homogeneous electric field.

1. Introduction

Consider the Schrödinger operator of a particle in a weak homogeneous electric field

$$P(\beta) = P_0(\beta) + V(x), \quad P_0(\beta) = -\Delta + \beta x_1, \tag{1.1}$$

where $x = (x_1, x') \in \mathbb{R} \times \mathbb{R}^{n-1}$ and $\beta > 0$ is a small parameter proportional to the field strength. The resonances of $P(\beta)$ have been studied by several authors ([4, 6, 8, 14, 17]). In particular, in [14, 18], upper bounds on the width of resonances were given in the multidimensional case and in [6, 19] the asymptotic behavior of the width was analysed. On the other hand, time-delay in scattering theory for the pair $(P_0(\beta), P(\beta))$ has been studied in [12, 13]. Many papers of the physics literature affirm an intrinsic relation between the width of resonance and the time-delay. If V is a spherically symmetric potential, it was formally derived in [11] that in a subspace of fixed angular momentum the time-delay near resonance energy is approximately equal to the inverse of the width.

The main purpose of this paper is to rigorously prove this relation for Schrödinger operators with Stark effect in the weak field limit. For the analogous problem in the semiclassical limit we refer to [5, 10].

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