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p-Adic String Compactified on a Torus

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Abstract. $U(1)^{\times D}$ model with the Villain action on a g-loop generalization F_g of the Bruhat-Tits tree for the p-adic linear group $GL(2, \mathbb{Q}_p)$ is considered. All correlation functions and the statistical sum are calculated. We compute also the averages of these correlation functions for N vertices attached to the boundary of F_g. When the compactification radius tends to infinity the averages provide the g-loop N-point amplitudes of the uncompactified p-adic string theory, in particular for $g=0$ the Freund-Olson amplitudes.

1. Introduction

The idea of a non-archimedean string proposed in the papers [1–4] has stimulated great activity in this field [5–9]. The authors of [3–4] have interpreted bosonic string amplitudes at the tree level of perturbation theory over the non-archimedean number field $\mathbb{Q}_p$ as integrals of some combinations of multiplicative characters over $\mathbb{Q}_p$ (it is very close logically to the definition of the corresponding amplitudes for the usual open string over the real number field $\mathbb{R}$). In refs. [8] these p-adic amplitudes were produced from some non-local scalar field theory on $\mathbb{Q}_p$. Then the local formulation was given [9] which is actually more similar to the archimedean (Polyakov's) one. In the papers [9–10] the connection was established between p-adic string amplitudes and the Gaussian model on the Bruhat-Tits tree [11–14]. The Bruhat-Tits tree T is manifestly determined to be the connected infinite graph with no loops, each vertex of T being connected with exactly $p+1$ neighbour vertices by links. The *branch* B_z is defined to be a connected subtree with the only boundary vertex z of the graph $T \backslash B_z$ in the interior of T. By definition, the branch contains no cycles. A g-loop generalization of the p-adic string theory is given by the theory on the generalized tree F_g. It consists of a finite connected graph F_g^R with g independent loops, which is called a *reduced graph*, the branches B_x, $x \in F_g^R$, and each vertex is connected by links with exactly $p+1$ nearest neighbours (for every link two endpoints of which are identified with a vertex, we include the vertex itself twice into the number of its nearest neighbours). If the vertex $x \in F_g^R$ has only one nearest neighbour $y \in F_g^R$, $x \neq y$, then p branches B_x and