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A Theorem on Ergodicity of Two-Dimensional Hyperbolic Billiards

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Abstract. Ergodicity of two-dimensional billiards which satisfy some general conditions is proved. This theorem is applied to one concrete class of billiards that contains, in particular, billiards in the "stadium."

Dynamic systems of the billiard type (or simply billiards) belong to one of the most popular and best investigated class of hyperbolic dynamical systems with singularities. Hyperbolicity leads to entailing strong stochastic properties of the system such as positivity of Kolmogorov-Sinai entropy, existence of ergodic components of positive measure etc. Nevertheless the usual proof (attributed to Hopf [H]) of the ergodicity of smooth hyperbolic dynamic systems (for instance Anosov systems) cannot be applied to billiards in view of the presence of singularities. Sinai pointed out first (and proved it in [S1] for Sinai billiards on $\mathbb{T}^2$) a new statement that gives the possibility of applying Hopf's idea for the proof of ergodicity of hyperbolic dynamic systems with singularities. In [BS] the proof of this statement was simplified and extended to a wider class of Sinai billiards. The corresponding assertion was named in [BS] the main theorem of the theory of Sinai billiards. Later [B1] the same assertion was used for the proof of ergodicity of billiards in domains with focusing components of the boundary. Therefore it is now called the main theorem of the theory of billards with hyperbolic behavior.

Sinai proposed in [S2, S3] a more general method for the proof of this theorem. Making use of this method it was possible in [SC] to prove the ergodicity of some classes of semidispersing billiards. Especially one has to mention the proof of ergodicity of the system of three billiard balls on the d-dimensional torus [KSS1] which was recently obtained by Kramli, Simanyi, Szász [KSS2]. The authors of [KSS1] used a modified version of the main theorem which they called a "transversal" fundamental theorem for semi-dispersing billiards [KSS2]. Here

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