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Determinants of Laplace-like Operators on Riemann Surfaces

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Abstract. We calculate determinants of second order partial differential operators defined on Riemann surfaces of genus greater than one using a relation between Selberg's zeta function and functional determinants. In addition, we perform a calculation of these determinants directly using Selberg's trace formula, and compare our results with previous computations which followed the latter route.

1. Introduction

Since Polyakov [1] introduced his geometric, covariant approach to string perturbation theory, the question of computing functional determinants of Laplace-like differential operators on compact Riemann surfaces has gained some attention. As it is well known in string perturbation theory (see e.g. [2, 4–6, 10, 15]), the functional integral representing the string partition function may be reduced to a finite dimensional integral over moduli (or super-moduli) space, where the integrand can be expressed in terms of some of the determinants considered here.

Several authors [2–9] have evaluated the principal dependence of these determinants on Selberg's zeta function for the most interesting case of surfaces of genus greater than one. However, to our knowledge only D'Hoker and Phong [3, 4] completed the computations in determining the full answer for operators acting on tensor and spinor-tensor fields of arbitrary weight. In the following we present an alternative and more straightforward calculation of those determinants. The commutation relations for the relevant first order differential operators allow us to derive the spectra of the Laplace-like operators of arbitrary weight recursively from those of lowest weight. This gives us the opportunity of setting up a closed formula for the determinants of all those operators and for all genera. For the case of constant-curvature surfaces of genus greater than one a product representation of Selberg's zeta function involving determinant functions was obtained in [7–9]. This enables us to derive explicit expressions for the desired determinants, including all constants. Alternatively, we perform a direct calcula-